Quadrilateral

Q1.: The angle of the quadrilateral are in the ratio 3:5:9:13. Find all the angle of quadrilateral.

Sol: Let the angles be $3x^\circ, 5x^\circ, 9x^\circ$ and $13x^\circ$

Then $3x^\circ + 5x^\circ + 9x^\circ + 13x = 360^\circ$ [Sum of angles of Quadrilateral $= 360^\circ$]

$\Rightarrow 30x = 360^\circ$

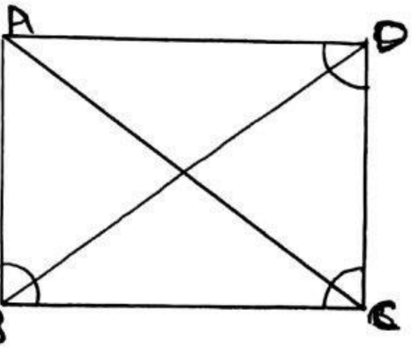
$x = \frac{360}{30} = \cancel{36}^\circ\ 12^\circ$

$\therefore$ The angles are $3 \times 12 = 36^\circ$, $5 \times 12 = 60^\circ$, $9 \times 12 = 108^\circ$, $13 \times 12 = 156^\circ$.

Q2: If the diagonals of parallelogram are equal, then show that it is a rectangle.

Sol: Given: A parallelogram ABCD in which $AC = BD$.

To prove: ABCD is a rectangle



Proof: In ΔABC and DCB.

$AB = CD$ (opp. side of ||gram)

$BC = BC$ (Common)

$AC = BD$ (Given)

$\Delta ABC \cong \Delta DCB$ [By SSS]

$\Rightarrow \angle ABC = \angle DCB$ [By CPCT]

or $\angle B = \angle C$

Also $\angle B + \angle C = 180^\circ$ [co-interior angle] [$AB \parallel CD$]

$\Rightarrow \angle B + \angle B = 180$

$\Rightarrow 2\angle B = 180$

$\Rightarrow \angle B = \frac{180}{2} = 90^\circ$

$\angle B = \angle C = 90^\circ$

ABCD is a ||gram. one of whose angle is 90°

Hence, ABCD is a rectangle.

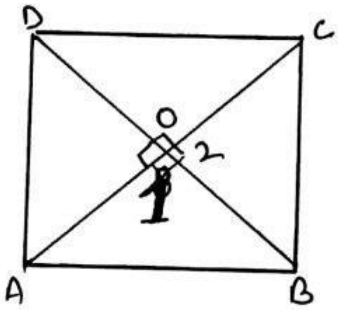
Ques3: Show that if the diagonal of a quad. bisect each other at right angles, then it is a rhombus.

Sol: Given: A quad ABCD in which diagonal AC and BD intersect O such that

$OA = OC$, $OB = OD$ and $AC \perp BD$

To prove: ABCD is a rhombus.

Proof: In ΔAOB and ΔBOC



$AO = OC$ (Given)

$\angle 1 = \angle 2$ (each 90°)

$BO = BO$ (common)

$\therefore \Delta AOB \cong \Delta BOC$. (By SAS)

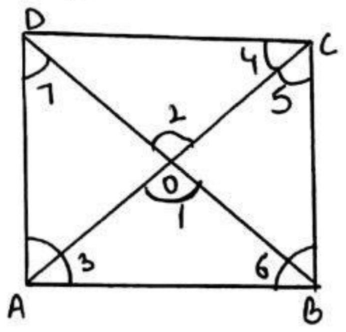
$AB = BC$. (By CPCT)

Hence, ABCD is a rhombus.

[∵ If the diagonal of quad bisect each other, then it is a ||gram and opposite side of ||gram]. #

Ques 4: Show that the diagonal of square are equal and bisect each other at right angles.

Sol: Given: ABCD is a square.



To prove: $AC = BD$

$AC \perp BD$

$AO = OC$, $OB = OD$.

Proof: In ΔABC and ΔBAD.

$AB = AB$

$BC = AD$ (Side of square)

$\angle ABC = \angle BAD$ (each 90°)

$\Delta ABC \cong \Delta BAD$ (By SAS) $\Rightarrow AC = BD$ (By CPCT)

In ΔAOB and COD

$AB = CD$ (side of square)

$\angle 1 = \angle 2$ (V.O.A)

$\angle 3 = \angle 4$ (A.I.A)

$\Delta AOB \cong \Delta COD$. (By ASA)

$\Rightarrow AO = OC$ # and $OB = OD$ # (By CPCT)

In ΔABC

$\angle 3 + \angle B + \angle 5 = 180^\circ$

$\Rightarrow \angle 3 + 90^\circ + \angle 3 = 180^\circ$ $\left[\begin{array}{l} AB = BC \\ \Rightarrow \angle 3 = \angle 5 \end{array}\right]$

$\Rightarrow 2\angle 3 = 180 - 90$

$\Rightarrow \angle 3 = \frac{90}{2} = 45^\circ$

$\Rightarrow \angle 3 = \angle 5 = 45^\circ$

Similarly $\angle 6 = \angle 7 = 45^\circ$

In ΔAOB

$\angle 3 + \angle 1 + \angle 6 = 180^\circ$

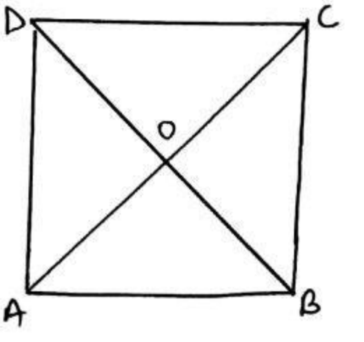
$45^\circ + \angle 1 + 45^\circ = 180$

$\angle 1 = 180 - 45 - 45 = 90^\circ$

$\angle BOC = 90^\circ \Rightarrow BO \perp AC$.

$\Rightarrow BD \perp AC$ #

Ques 5: Show that if the diagonals of a quad. are equal and bisect each other at rt. angles, then it is a square



Given: A quad. ABCD, in which

$AC = BD$. and $OA = OC$

$BD \perp AC$. $OB = OD$

To prove: ABCD is a square.

Proof: ABCD is a quad. whose diagonal bisect each other, so it is a ||gram.

Also, its diagonal bisect each other at right angle.

$\therefore$ ABCD is a rhombus.

$\Rightarrow AB = BC = CD = DA.$

In ΔABC and ΔBAD.

$AB = AB$ (common)

$BC = AD$ (Proved above)

$AC = BD$ (Given)

$\therefore \Delta ABC \cong \Delta BAD$ (By SSS)

$\Rightarrow \angle ABC = \angle BAD$ (By CPCT)

$\Rightarrow \angle A = \angle B = x$ (let)

$\angle A + \angle B = 180^\circ$

$x + x = 180$

$2x = 180$

$x = \frac{180}{2} = 90^\circ$

$\angle A = \angle B = \angle C = \angle D = 90^\circ$ [opp. ~~side~~ angle of ||gram]

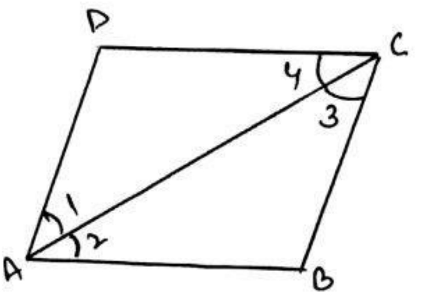
$\Rightarrow$ ABCD is a rhombus whose angle are of 90° each

Hence ABCD is a square.

Ques 6:- Diagonal AC of parallelogram ABCD. bisect $\angle A$

(i) it bisect $\angle C$

(ii) ABCD is a rhombus.



Given: In ||gram ABCD

$\angle 1 = \angle 2.$

Proof: $AB \parallel CD$ and $BC \parallel AD$

$\Rightarrow \left.\begin{array}{l}\angle 1 = \angle 3 - (1)\\ \angle 2 = \angle 4 - (2)\end{array}\right]$ [A.I.A]

Given $\angle 1 = \angle 2$ - (3)

From (1) (2) (3) $\Rightarrow \angle 3 = \angle 4.$

$\therefore$ AC bisect $\angle C$.

In $\triangle ABC$ and $\triangle CDA$ From (1) (2) (3)

$AC = AC$

$\angle 1 = \angle 2 = \angle 3 = \angle 4$

In $\triangle ABC$, $\angle 2 = \angle 3$

$\Rightarrow AB = BC$

In $\triangle ADC$, $\angle 1 = \angle 4$

$\Rightarrow AD = CD$.

Also, ABCD is a ||gm.

$AB = CD, AD = BC$.

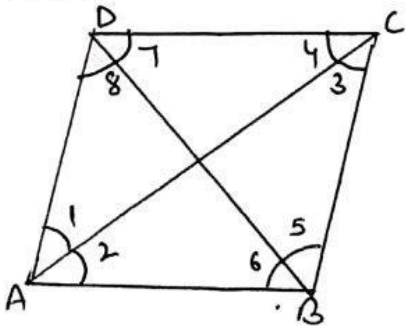
$\therefore AB = BC = CD = DA$

Hence, ABCD is a rhombus.

Q7 ABCD is a rhombus. Show that the diagonal AC bisect $\angle A$ as well as $\angle C$ and diagonal BD bisect $\angle B$ as well as $\angle D$

Sol: Given: ABCD is a rhombus. i.e

$AB = BC = CD = DA$



To prove: $\angle 1 = \angle 2$

$\angle 3 = \angle 4$

$\angle 5 = \angle 6$

$\angle 7 = \angle 8$

In $\triangle ABC$ and CDA

$AB = AD$ (Given)

$AC = AC$ (common)

$BC = DA$ (Given)

$\triangle ABC \cong \triangle CDA$ [By SSS]

$\Rightarrow \angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [By CPCT]

Similarly $\angle 6 = \angle 5$ and $\angle 7 = \angle 8$.

Hence diagonal AC bisect $\angle A$ and $\angle C$ and diagonal BD bisect $\angle B$ and $\angle D$.

Q8: ABCD is a rectangle in which diagonal AC bisect $\angle A$ as well as $\angle C$. Show that (i) ABCD is a square.
(ii) Diagonal BD bisect $\angle B$ as well as $\angle D$.

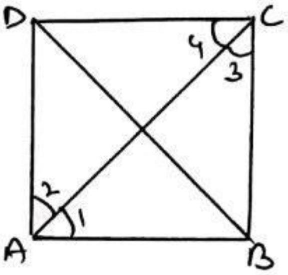
Sol: Given : ABCD is a rectangle.
AC bisect $\angle A$ and $\angle D$.

To prove : (i) ABCD is a square.
(ii) Diagonal BD bisect $\angle B$ as well as $\angle D$.

Proof : (i) Since AC bisect $\angle A$ and $\angle C$.

$\angle 1 = \angle 2 = \angle 3 = \angle 4$ $\left[\because \text{each } \frac{90}{2} = 45^\circ\right]$

In ΔABC



$\angle 1 = \angle 3$

$\Rightarrow AB = BC$. (side opp. to equal angle)

In rectangle opp. side are equal

$\therefore AB = BC = CD = DA$

Thus, ABCD is a square.

(ii) In square, diagonal bisect the angle D and B.
So, BD bisect $\angle B$ as well as $\angle D$.

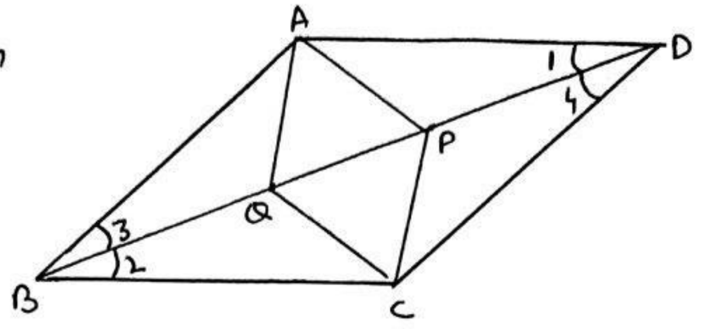
Q9: In a parallelogram ABCD, two point P and Q are taken on diagonal BD such that DP = BQ. Show that

(i) DAPD $\cong$ DCQB
(ii) AP = CQ
(iii) $\Delta AQB = \Delta CPD$.
(iv) AQ = CP
(v) APCQ is a parallelogram.

Given : ABCD is a ||gram

P and Q are point on diagonal BD

Such that DP = BQ



To prove : ______

Proof :

(I) In ΔAPD and ΔCQB.

$AD = BC$ (opp. side of ||gram)

$DP = BQ$ (Given)

$\angle 1 = \angle 2$ (A.I.A)

$\Delta APD \cong \Delta CQB$ (By SAS)

(II) $\therefore AP = CQ$ [CPCT]

(iii) In ΔAQB and ΔCPD, we have.

$AB = CD$ [opp. side of ||gram]

$DP = BQ$ (Given)

$\angle 3 = \angle 4$ [A.I.A]

$\Delta AQB \cong \Delta CPD$ [By SAS]

(IV) $AQ = CP$ [By CPCT]

(V) Since in APCQ, opposite side are equal

$\therefore$ it is a ||gram.

Q10. ABCD is a ||gram and AP and CQ are ⊥ar from vertices A and C on diagonal BD respectively. Show that (I) $\Delta APB \cong \Delta CQD$.

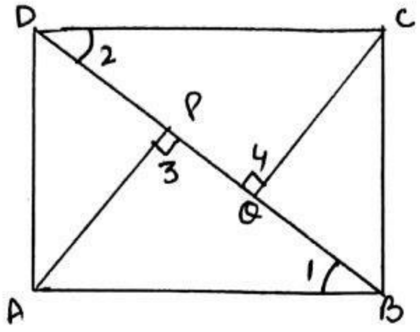
(II) $AP = CQ$.

Sol :.

Given ABCD is a ||gram

AP and CQ are ⊥ar from vertices A and C. on diagonal BD

Proof: In ΔAPB and ΔCQD



$\angle 1 = \angle 2$ (A.I.A)

$AB = CD$ (opp. side of ||gram)

$\angle 3 = \angle 4$ (each $90°$)

$\Delta APB \cong \Delta CQD$ (ASA).

So $AP = CQ$. [Proved]
[CPCT]

Q11. In ΔABC and ΔDEF, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B, C are joined to vertices D, E, F. Show that

(1) Quad ABED is a ||gram

(2) Quad BEFC is a ||gram

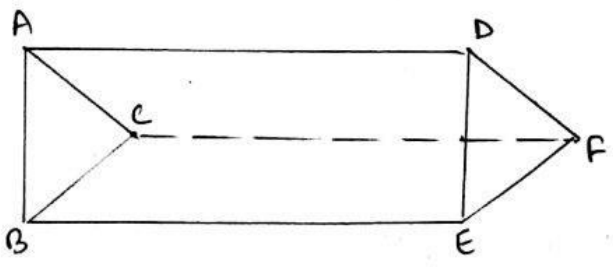
(3) $AD \parallel CF$ and $AD = CF$.

(4) Quad. ACFD is a ||gram

(5) $AC = DF$

(6) $\Delta ABC \cong DEF$.

Sol :- Given : Two ΔABC and ΔDEF such that



$AB = DE$ and $AB \parallel DE$

$BC = EF$ and $BC \parallel EF$

To prove: See as above

Proof: (1) In quad ABED, $AB = DE$ and $AB \parallel DE$
one pair of opp. side is equal and ||el
$\Rightarrow$ ABED is a ||gram.

(ii) In quad BEFC, we have $BC = EF$ and $BC \parallel EF$

one pair of opp. side is equal and ||el.

$\therefore$ BEFC is a ||gram.

Now (iii) $AD = BE$ and $AD \parallel BE$ — (1)

[$\because$ ABED is a ||gram]

and $CF = BE$ and $CF \parallel BE$ — (2)

[$\because$ BEFC is a ||gram]

From (1) and (2) $AD = CF$ and $AD \parallel CF$.

$\Rightarrow$ one pair of opp. side is equal and ||el

$\Rightarrow$ ACFD is a ||gram. [Proved IV]

(V) Since ACFD is a parallelogram.

$AC = DF$ [opp. side of ||gm ACFD]

(VI) In ΔABC and ΔDEF, we have.

$AB = DE$ (opp. side of ||gram)

$BC = \cancel{B}EF$. "

$CA = FD$ "

$\therefore \Delta ABC \cong \Delta DEF$ (By SSS).

Q12:- ABCD is a trapazium in which $AB \parallel CD$ and $AD = BC$. Show that

(I) $\cancel{BA} = \cancel{BD}$ $\angle A = \angle B$

(II) $\angle C = \angle D$

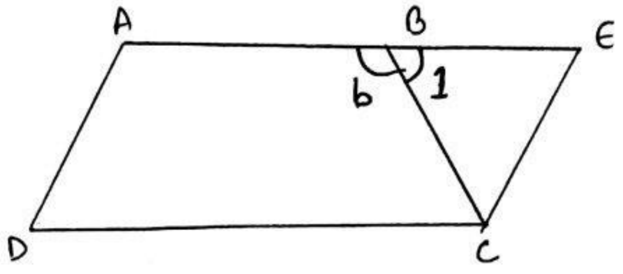
(III) $\Delta ABC = \Delta BAD$.

(IV) Diagonal $AC =$ Diagonal BD.

Given : ABCD is a trapazium in which AB||CD
and AD = BC

<u>To prove</u> : See as above . <u>Const</u>:- Produce AB and draw a line CE||AD

Proof :



Since AB||DC -

$\Rightarrow$ AE||DC —(i)

and AD||CE — (ii)

$\Rightarrow$ ADCE is a ||gram. [opp. side of ||gram are ||el]

$\angle A + \angle E = 180$ — (iii) [consecutive interior angle]

$\angle B + \angle 1 = 180^\circ$ — (4) [linear pair]

AD = CE [opp. side of ||gram are equal] — (5)

BC = CE — (6) [Given]

From (5) and (6)

AD = BC

$\Rightarrow \angle 1 = \angle E$ — (7) [angle opp. to equal side are equal]

From (iii) and (7)

$\angle A + \angle 1 = 180$ — (8)

From (4) and (8)

$\angle B + \angle 1 = \angle A + \angle 1$

$\angle B = \angle A$ [Proved.]

(ii) $\angle A + \angle D = 180^\circ$

$\angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + \angle D = \angle B + \angle C$

$\Rightarrow \angle D = \angle C$ [$\because \angle A = \angle B$]

(iii) In ΔABC and ΔBAD

AD = BC (Given)

$\angle A = \angle B$ [Proved]

$AB = AB$ (common)

$\therefore \triangle ABC \cong \triangle BAD$ [By ASA]

(iv) diagonal AC = diagonal BD [By CPCT].

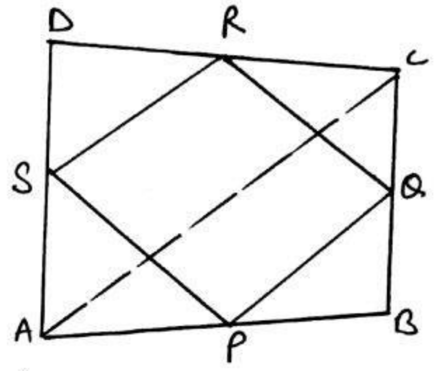
Exercise : 8.2

Q1. ABCD is a quadrilateral in which P, Q, R and S are mid point of side AB, BC, CD and DA respectively AC is a diagonal. Show that

(i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$.

(ii) $PQ = SR$

(iii) PQRS is a parallelogram.



Given : See as above.

To prove : See as above

Proof :- In $\triangle ABC$, P is the mid point of AB and Q is the mid point of BC.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ — (1) [By Mid point Theorem]

In $\triangle ADC$, R is the mid point of CD and S is the mid point of AD.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ — (2) [Proved (i)]

[By Mid point theorem]

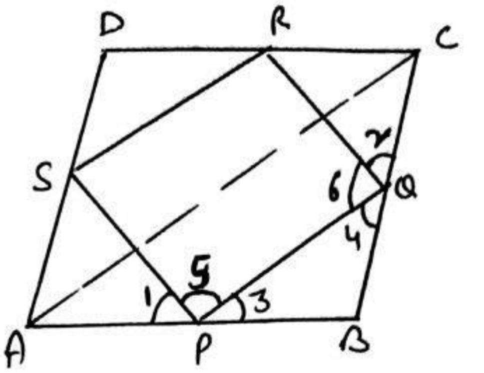
(ii) From (1) and (II)

$PQ \parallel SR$ and $PQ = SR$

(iii) Now in quadrilateral PQRS, its one pair of opposite side is equal and $\parallel$el.

$\therefore$ PQRS is a parallelogram.

Q2. ABCD is a rhombus and P, Q, R, S are the mid point of the sides AB, BC, CD and DA respectively. Show that the quad. PQRS is a rectangle.



Sol: Given :-

To prove : PQRS is a rectangle.

Const. Join AC.

Proof :- In ΔABC, P and Q are mid pt. of side AB and BC.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$ —(1)

In ΔADC, R and S are mid point of side CD and AD.

$\therefore SR \parallel AC$ and $SR = \frac{1}{2} AC$ —(2)

From (1) and (ii)

$PQ \parallel RS$ and $PQ = SR$

In quad PQRS, one pair of opposite side is equal and parallel.

$\therefore$ PQRS is a parallelogram.

$\therefore AB = BC$

$\frac{1}{2} AB = \frac{1}{2} BC$

$\Rightarrow PQ = BQ$

$\Rightarrow \angle 3 = \angle 4$ — ③

[angle opp. to equal side are equal].

In ΔAPS and ΔCQR .

$AP = CQ$ [Half of equal side AB, BC and AD, CD]

$AS = CR$

$PS = QR$ [opp. side of ||gram]

$\Delta APS \cong \Delta CQR$ [SSS]

$\angle 1 = \angle 2$ (By CPCT) — (4)

$\angle 1 + \angle 5 + \angle 3 = 180°$ — (5)

$\angle 2 + \angle 4 + \angle 6 = 180$ — (6)

$\angle 1 + \angle 5 + \angle 3 = \angle 2 + \angle 4 + \angle 6$

$\Rightarrow \angle 5 = \angle 6$ [$\because \angle 1 = \angle 2$, $\angle 3 = \angle 4$]

(or) $\Rightarrow \angle SPQ = \angle PQR$

Also $SP \parallel RQ$.

$\angle SPQ + \angle PQR = 180°$

$\angle SPQ + \angle SPQ = 180$

$2\angle SPQ = 180$

$\angle SPQ = \frac{180}{2} = 90°$

$\Rightarrow \angle SPQ = \angle PQR = 90°$

Thus, PQRS is a rectangle where one angle $\angle SPQ = 90°$ #

Q3:- ABCD is a rectangle and P, Q, R and S are mid point of the side AB, BC, CD, DA. Show that quadrilateral PQRS is a rhombus.

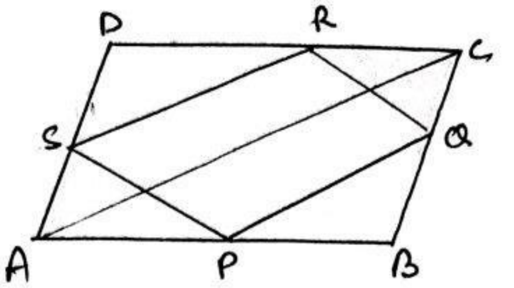
Sol:- Given :-

To prove: PQRS is a rhombus

Const: Join AC.

Given :-

To Proof : In ΔABC,



P and Q are mid point of side AB and BC.

$PQ = AC$, $PQ = \frac{1}{2} AC$ — (1)

Similarly in ΔADC, R and S are mid point of side CD and AD.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2} AC$. — (2)

From (2) and (1)

$PQ \parallel SR$ and $PQ = SR$. — (3)

Now in quad. PQRS, one pair of opposite side PQ and SR is parallel and equal.

$\therefore$ PQRS is a parallelogram — (4)

Now $AD = BC$

$\Rightarrow \frac{1}{2} AD = \frac{1}{2} BC$

$\Rightarrow SA\cancel{B} = BQ$.

In ΔAPS and ΔQBP

$\angle A = \angle B$ [each 90°]

$AS = BQ$

$AP = PB$ [P is the mid point]

$\Delta APS \cong \Delta QBP$

$\Rightarrow PS = PQ$ [CPCT] — (5)

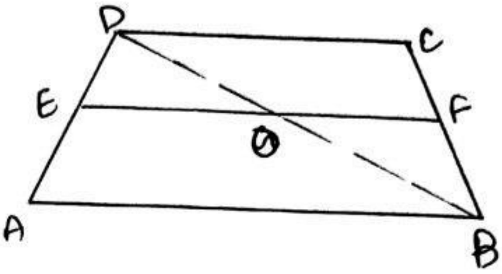
From (4) and (5), we get

$PS = PQ = QR = SR$.

$\Rightarrow$ PQRS is a rhombus

Q4:. ABCD is a trapazium in which AB||DC. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid point of B.

Sol:- In Δ DAB,



E is the mid point of AD.

EO||AB [$\because$ EF||AB]

$\therefore$ By converse of mid pt. theorem. O is the mid point of BD.

In ΔBCD. O is the mid point of BD.

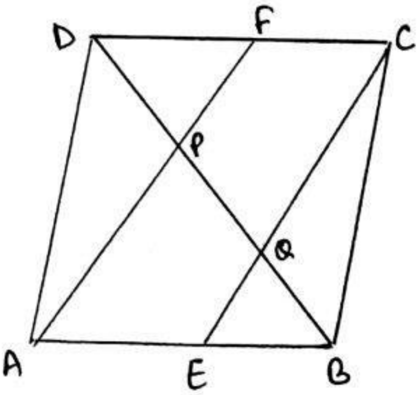
OF||CD.

$\therefore$ By converse of mid point theorem.

F is the mid point BC.

Ques 5: In a parallelogram ABCD, E and F are mid point of side AB and CD. Show that the line segment AF and EC intersect diagonal BD.

Sol:- To prove: BQ = QP = PD.



Proof: Since E and F are mid point of AB and CD.

$\Rightarrow AE = \frac{1}{2}AB$

and $CF = \frac{1}{2}DC$

But AB = CD and AB||CD.

$\therefore$ AE = CF and AE||CF.

$\Rightarrow$ AECF is a parallelogram.

[One pair of opp. side is ||el and equal]

In ΔBAP.

E is the mid point of AB

$EQ \parallel AP$.

$\Rightarrow$ Q is the mid point of PB

$\Rightarrow PQ = QB$. — (1)

Similarly, in ΔDQC.

P is the mid point of DQ

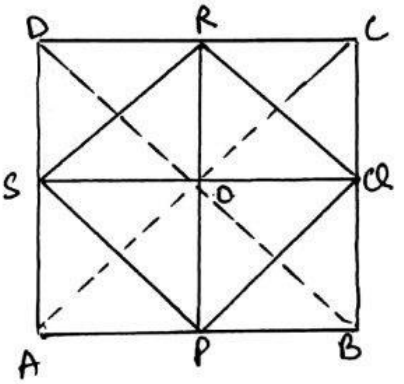
$DP = PQ$. — (2)

From (1) and (2)

$DP = PQ = QB$ #

Ques 6. Show that the line segment joining the mid point of opposite side of a quadrilateral bisect each other.

Sol:. Const. Join PQ, QR, RS, SP, AC, BD.



Proof: In ΔABC, P and Q are mid point of AB and BC.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2} AC$

In ACD, S and R are mid point of AD and CD.

$RS \parallel AC$ and $RS = \frac{1}{2} AC$

$\therefore PQ \parallel SR$ and $PQ = SR$.

Thus, a pair of opposite side of a quad. PQRS are parallel and equal.

$\therefore$ PQRS is a parallelogram.

And in parallelogram. diagonal bisect each other.

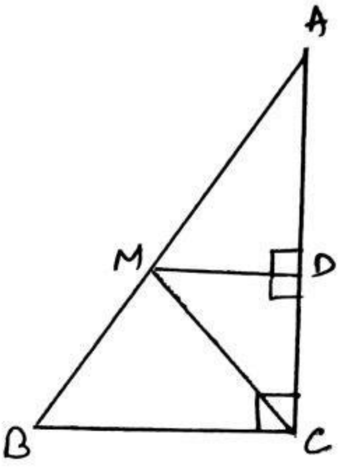
$\Rightarrow OP = OR$, $OQ = OS$.

Ques7. ABC is a triangle right angle at C. A line through the mid point M of hypotenuse AB and parallel to BC intersect AC at D. Show that :

(i) D is the mid point AC.

(ii) $MD \perp AC$.

(iii) $CM = MA = \frac{1}{2} AB$.



Proof:- In ΔABC, M is the mid point of AB and $MD \parallel BC$.

$\therefore$ D is the mid point of AC.

$\Rightarrow AD = DC$.

(ii) Since $MD \parallel BC$

$\angle ADM = \angle ACB = 90^\circ$ [Corresponding angles]

$\Rightarrow MD \perp AC$.

(iii) In ΔAMD and CMD.

$AD = CD$

$\angle ADM = \angle CDM$

$MD = MD$ (Common)

$\Delta AMD \cong \Delta CMD$.

$\Rightarrow MA = MC$. [By CPCT]

$MA = \frac{1}{2} AB$. Since M is the mid point of AC.

Hence $CM = MA = \frac{1}{2} AB$. #